Religious affiliation switching: The inevitable decline of organized religion

Designing and Constructing Models with Multi-agent Languages
EECS 472
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Contents

1 Introduction

People claiming no religious affiliation constitute the fastest growing religious minority in many countries, including the United States [1]. In fact, the religious "nones" are the only group growing in all 50 US states [2]. Although many scholars attribute the decline of religious affiliation to generational changes, roughly half of the US population changes religious affiliation at some point in their life, often several times [3]. This suggests that religious affiliation shift can be modeled as social group competition, with different religious groups competing for members [4]. Such models predict that religious coexistence is not a stable state; the unaffiliated group will grow until all organized religion has disappeared. Whether or not this comes to fruition, all people have a stake in the outcome.

2 Dynamical systems model

Abrams et al. split an ideal society into the mutually exclusive religiously affiliated and unaffiliated, with the fraction x belonging to the unaffiliated group and $y = 1 - x$ belonging to affiliated. Assuming that people only switch affiliations based on the fraction of people in each group and the perceived utility of the group, the dynamics of conversion can be modeled by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = yP_{yx}(x, u_x) - xP_{xy}(x, u_y), \tag{1}$$

where $P_{yx}(x, u_x)$ is the probability per unit time that an individual converts from religious to unaffiliated, $0 \leq u_x \leq 1$ is the perceived utility of being unaffiliated, and $u_y = 1 - u_x$ is the perceived utility of being affiliated. The authors further assume that (1) is symmetric under exchange of x and y and that no individual would switch to a group with no members (i.e. $P_{yx}(0, u_x) = 0$).

If the transition probabilities are smooth and monotonically increasing in both arguments, then there exist at most three fixed points with alternating stability. Assuming this model is a valid representation of religious affiliation switching, all available data suggest that the inevitable steady state is $x^* = 1$, or the extinction of religious affiliation. For specificity, the authors chose the power law $P_{yx}(x, u_x) = cx^a u_x$, where c and a are constants that scale time and determine the relative importance of x and u_x in attracting converts, respectively. For simplicity, the authors consider integer values of a, and the best fit to data occurs for $a = 1$. See Figure 1 for data and model fits.

The authors extend this model to binary networks of individuals (rather than all-to-all coupling) and allow for a continuous "religiosity" degree (rather than binary in or out of group), but the final state remains the same. As long as the network is not completely disconnected, only a time delay is introduced. See Figure 2.

3 Agent based model

Minimal continuous dynamical systems lend themselves well to rigorous analysis, but important details may be left out for simplicity. For instance, Abrams et al. ignore heterogeneity in religious affiliation utility and social desire for similarly affiliated friends. Agent based modeling is a natural way to test the robustness of the continuous model results under assumptions of heterogeneity and network adaptation. I implement the following model in NetLogo [5].

3.1 Agent properties

Each agent is a person in a community connected via a social network. Each person perceives a constant utility u_x of religious non-affiliation. An agent is either religiously affiliated or unaffiliated at any point in time, and it switches affiliation based on the fraction of friends who are affiliated and the perceived utility of affiliation. Agents also prefer a certain fraction of their friends to be similarly affiliated, so they form and break friendships to meet a minimum similarity desire (details below).

3.2 Model parameters

Parameter	Description
COMMUNITY-SIZE	The number of agents in the simulation.
INITIAL-X	Initial proportion of agents that are unaffiliated.
U-X	Mean of Gaussian distribution of unaffiliated utility.
U-X-VAR	Standard deviation of Gaussian distribution of unaffiliated utility.
TIME-SCALE	Time scale for switching rates.
AFFILIATION-POWER	Power determining relative importance of utility and affiliation proportion in attracting converts.
NETWORK?	If off, network is all-to-all. If on, network is spatially clustered.
MIN-FRIEND-SIMILARITY	Proportion of similarly affiliated friends desired.
NEW-FRIEND-CHANCE	Probability of a friendless agent making a similarly affiliated friend.
AVERAGE-NODE-DEGREE	Initial average number of agent friends.
MAX-FRIENDS	Number of friends required to stop looking for new friends.

3.3 Setup

Initially, COMMUNITY-SIZE agents are given a random unaffiliated utility sampled from a normal distribution with mean U-X and standard deviation U-X-VAR (taken to be 0 for initial tests). A proportion INITIAL-X of the population is unaffiliated and the rest are affiliated. To be consistent with all available data, INITIAL-X must be small ($\approx 5\%$). If NETWORK? is on, a spatially clustered network with AVERAGE-NODE-DEGREE is created, using code from [6].

3.4 Runtime

At each tick, affiliated agents do the following (similarly for unaffiliated agents):

- Switch to unaffiliated with probability TIME-SCALE $*u_x * x$^AFFILIATION-POWER, where x is the unaffiliated proportion of an agent's friends.
- If you switch, break enough friendships with random affiliated friends until your proportion of unaffiliated friends is at least MIN-FRIEND-SIMILARITY.
- If you have fewer than MAX-FRIENDS, make a new unaffiliated friend with probability NEW-FRIEND-CHANCE / $(100 * (\text{num-friends } + 1))$. The new friend should be the closest unaffiliated agent.

3.5 Model justification

Like all models, this model of religious affiliation switching uses several simplifying assumptions. First, I assume that religious affiliation switching can be modeled as group competition. This is reasonable because religious affiliation has less to do with belief and more to do with social belonging. In fact, most unaffiliated individuals believe in god, and a third consider religion to be an important part of their lives [7]. Taken with the fact that people often switch affiliations several times [3], religious affiliation shift can be modeled as social group competition, with different religious groups competing for members. I therefore use Abrams et al. model of group competition as a starting point for this agent based model.

Second, this model assumes that switching affiliations costs a person friends. When an agent switches from affiliated to unaffiliated, it loses affiliated friends up to a diversity tolerance MIN-FRIEND-SIMILARITY. Without imposing this cost to an agent's social connections, the agent can switch affiliations capriciously. This is inconsistent with the fact that people only switch affiliations a few times per lifetime, on average [3].

Third, I assume that people occasionally make friends with nearby individuals who are similarly affiliated. Because the network is spatially clustered, close agents are likely friends of friends; it makes more sense that people form friendships with individuals who are only a few degrees a separation away.

3.6 Model verification

I first verify that an agent based implementation of (1) with an all-to-all network replicates the approximately logarithmic growth of the unaffiliated group and the eventual extinction of the religiously affiliated. I set all agent unaffiliated utilities to $u_x = 0.65$ because that is the average utility for all 85 available datasets. See Figure 3.

I also verify that implementation of (1) with social network replicates the approximately logarithmic growth of the unaffiliated group with a time delay to the extinction of the religiously affiliated. As AVERGE-NODE-DEGREE decreases, the time delay increases, which is consistent with results presented in [4]. For small average node degree (< 5), the affiliated group no longer goes extinct because small clusters are not connected. See Figure 4.

3.7 Model behavior

After confirming that the agent based model (with specific settings) produces the same results as the dynamical systems model, I test the robustness of the model predictions.

3.7.1 Utility heterogeneity

One of the main assumptions of the continuous model is that the entire population perceives that religious affiliation has the same utility. While this assumption greatly simplifies analysis, it is much more realistic to assume that people perceive a wide range of utility in affiliating themselves with a religious organization. Due to the ubiquity of normal distributions in nature, I assume that unaffiliated utility follows a Gaussian distribution with mean U-X between 0.53 and 0.72, as indicated by world-wide data [4].

For U-X $= 0.65$ (the average for all available data), the model results are robust to small utility standard deviations U-X-VAR. However, as the standard deviation of utility increases such that a significant number of people have $u_x < 0.5$, then the system stabilizes with a small fraction of affiliated individuals. See Figure 5 for the equilibrium state for various standard deviations of utility.

In other words, introduction of a wide range of perceived utility implies that the two groups can coexist long term. See Figure 6 for an example simulation with large standard deviation of utility.

3.7.2 Exponent of power law

Abrams et al. found that the exponent $a = 1$ minimized the model error using real-world data, but the authors only checked integer values of a for simplicity. While the model is robust to small changes in a, qualitative differences emerge when the exponent changes too much. For $a > 1$ the unaffiliated population goes extinct, and for $a < 1$ the two groups can coexist. See Figure 7. Sensitivity analysis shows that the final proportion of unaffiliated agents remains well above 0.9 for $a < 1$, even for moderate utility standard deviation. Therefore the model is robust to changes in $a < 1$.

3.7.3 Affiliation-dependent network

One of the assumptions of the continuous model (1) is that the network is static, but it's more realistic to assume that people lose and make friends especially when they switch affiliations. At each time step, people switch affiliations according the previous transition probabilities. However, if an agent switches from affiliated to unaffiliated in that time step, it calculates what proportion of its friends are unaffiliated. If that proportion is less than the MIN-FRIEND-SIMILARITY desired by all agents, the agent break ties with random affiliated friends until the MIN-FRIEND-SIMILARITY proportion is reached. If agents want most of their friends to have similar affiliations, then the unaffiliated and affiliated groups can coexist. The system stabilizes when the affiliated agents either form their own separate cluster or disconnect from the network entirely. See Figure 8 for the equilibrium state for various minimum friend similarities.

See Figure 9 for an example simulation with high minimum friend similarity. Note that the network separates into chain of friends, with many individuals separated from the network entirely. This does not resemble a real social network because people cannot make new friends.

In addition to losing friends after switching from affiliated to unaffiliated, each agent might gain a new unaffiliated friend with NEW-FRIEND-CHANCE. If an agent makes a new friend, it choses the closest similarly affiliated individual. This agent is most likely a friend of a friend due to proximity. As long as the chance of making new friends is not exactly 0, then the proportion of unaffiliated agents at equilibrium is approximately the same. See Figure 10 for sensitivity analysis of the NEW-FRIEND-CHANCE parameter.

4 Model validation

Available data suggest that the unaffiliated group grows from a small proportion of the population at an increasing rate. Most time series end before the unaffiliated group exceeds 50% of the population, so the final state is unknown. See Figure 1. Averaged over many simulations, the agent based model should replicate this behavior. Ideally, the real data will fit within approximately one standard deviation of many simulations.

For illustrative purposes, I present three national census datasets (Netherlands, Czech Republic, and New Zealand) with model predictions. I vary only the average u_x when fitting the data to the model and hold all other parameters constant across all datasets. Note that the model does not predict extinction of the religiously affiliated group for any dataset.

5 HubNet model

I built a generic social group competition model using HubNet where users choose a group to join based on the current membership and utility of the group. Though I haven't tested the model with real players, it would be interesting to see if the same transition functions emerge from this abstraction of group competition. Large colored patches represent different group affiliations, and users move to specific patches based on provided utilities (varying across users) and the number of other agents on the same patch. See Figure 12 for the observer and player views.

6 Possible model extensions

Extensions of this model could incorporate birth, death, and immigration into the model. Different affiliations may have different birth and death rates. Immigrants may have different religious affiliations from the current population. In addition, multiple affiliations could be introduced, rather than the binary affiliated and unaffiliated.

An extension to the HubNet model would be imposing network structure. In that case, non-neighboring agents would be invisible to players. It would also be interesting to add more groups to the game.

7 Conclusion

This agent based model of religious affiliation replicates the continuous dynamical systems model on a network under the specified conditions. Specifically, the agent based model predicts logarithmic growth of the unaffiliated group and the eventual extinction of the affiliated group. The early dynamics of the model are consistent with real-world data, but the world is far from religious equilibrium. It is therefore impossible to use real-world data to select which of the models best describes religious shift or what the eventual outcome will be.

However, the extinction of the religiously affiliated group is not inevitable as proposed in [4]. Adding only heterogeneity of perceived religious affiliation utility can result in coexistence of the groups long term. Coexistence of the affiliated and unaffiliated groups is also possible when the people are connected in a social network, only looking to their friends to decide their affiliation. When people break ties with differently affiliated people and make friends with similarly affiliated people, the unaffiliated group becomes a clear majority, but a small number of affiliated members survive. Therefore the extinction of religious affiliation is not inevitable.

References

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8 Figures

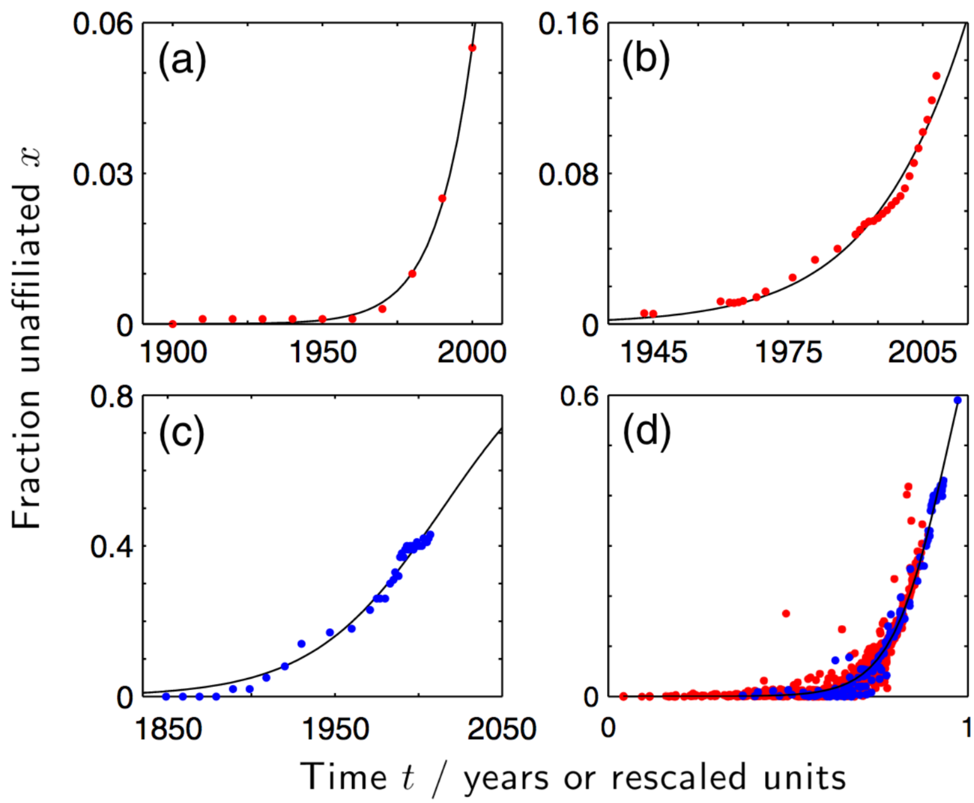

Figure 1: Fraction of population religiously unaffiliated versus time for (a) Schwyz Canton in Switzerland, (b) the autonomous Aland islands region of Finland, (c) the Netherlands, and (d) all 85 data sets. Dots indicate data points from census surveys; red dots correspond to regions within countries and blue dots to entire countries. Black lines indicate fits to model (1). For (a)-(c), relative utilities for the religiously unaffiliated populations as determined by dynamical system model fits were $u_x = 0.70, 0.63, 0.56$. For (d) time has been rescaled so data sets lie on top of one another and the solution curve with $u_x = 0.65$. Figure copied from [4].

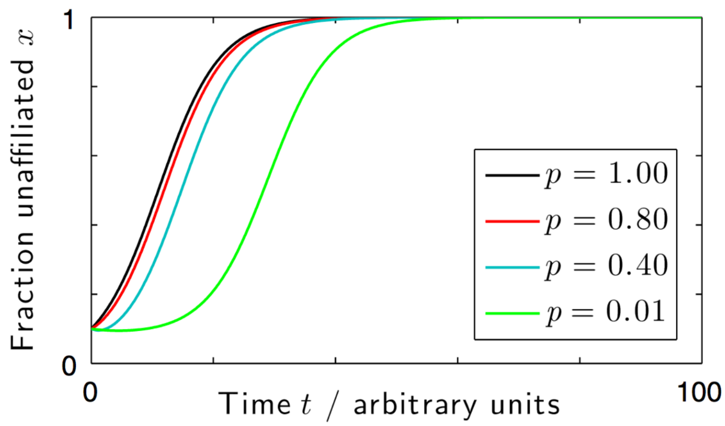

Figure 2: Fraction of population religiously unaffiliated versus time for varying degrees of network connectivity. The parameter p indicates the probability of a link between members of two all-to-all clusters. As the network becomes less connected, the time to religious extinction increases. Figure copied from [4].

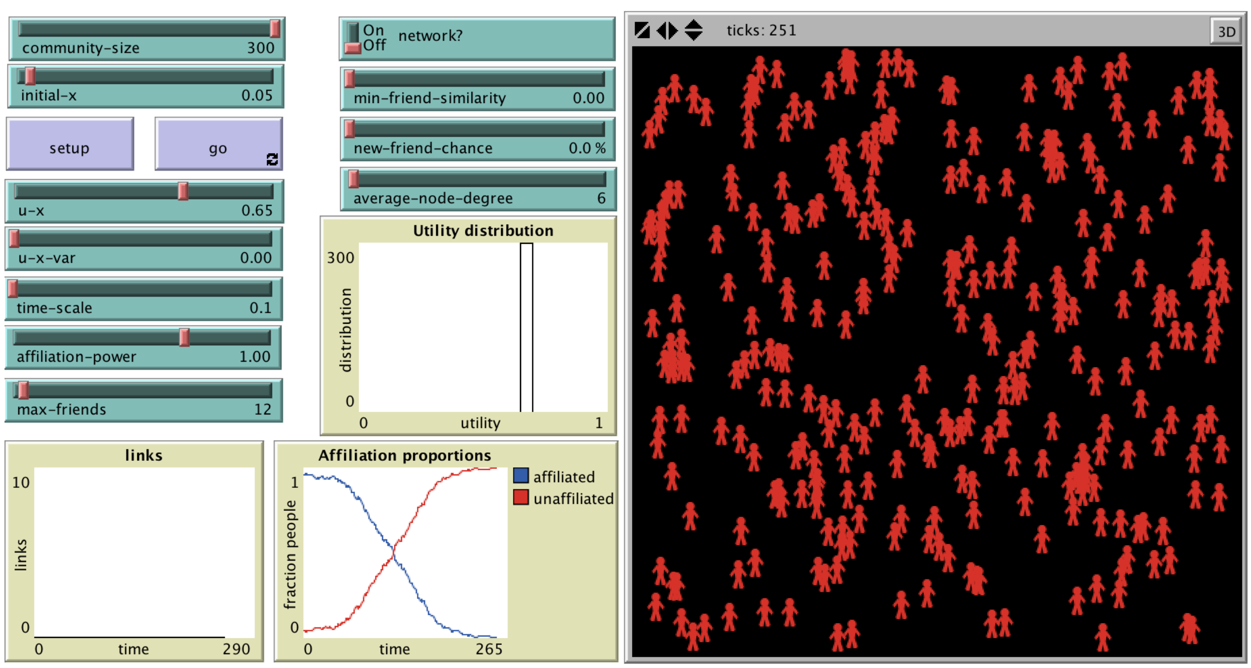

Figure 3: Verification that the agent based implementation of (1) on an all-to-all network replicates the logarithmic growth of an initially small unaffiliated population. For any uniform unaffiliated utility u_x exceeding 0.5, the religiously affiliated group will convert to unaffiliated.

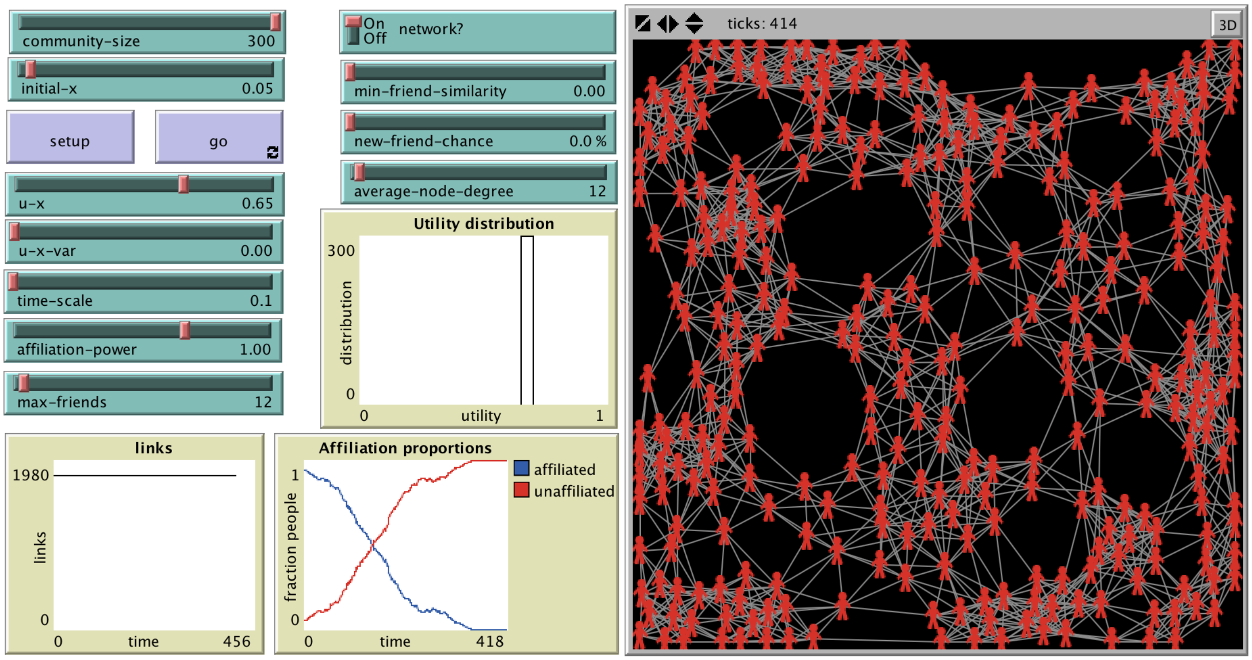

Figure 4: Verification that the agent based implementation of (1) on a network replicates the logarithmic growth of an initially small unaffiliated population. For any uniform unaffiliated utility u_x exceeding 0.5, the religiously affiliated group will convert to unaffiliated, but with a time delay from the all-to-all network.

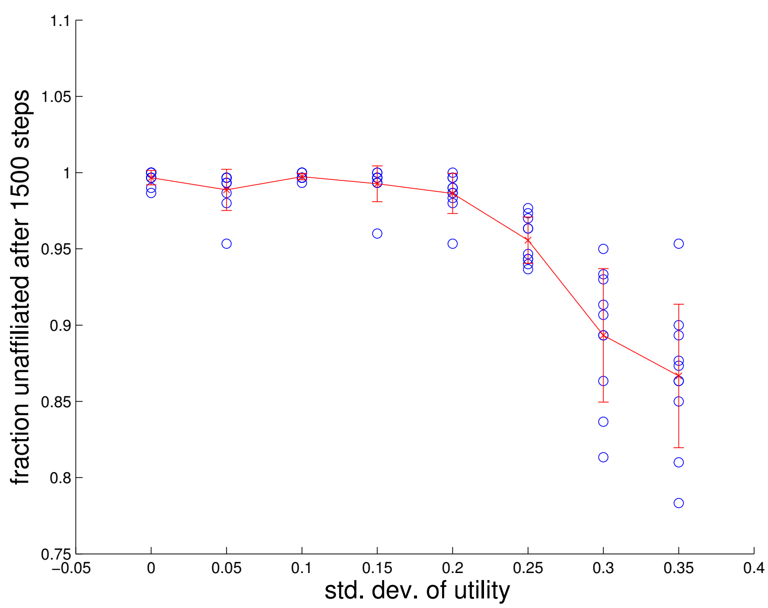

Figure 5: For a distribution of u_x centered about 0.65, the fraction of unaffiliated individuals at equilibrium decreases as the standard deviation of utility increases. Blue dots indicate individual simulations, red x's indicate the average of the 10 simulations, and the red error bars indicate $\pm$ one standard deviation.

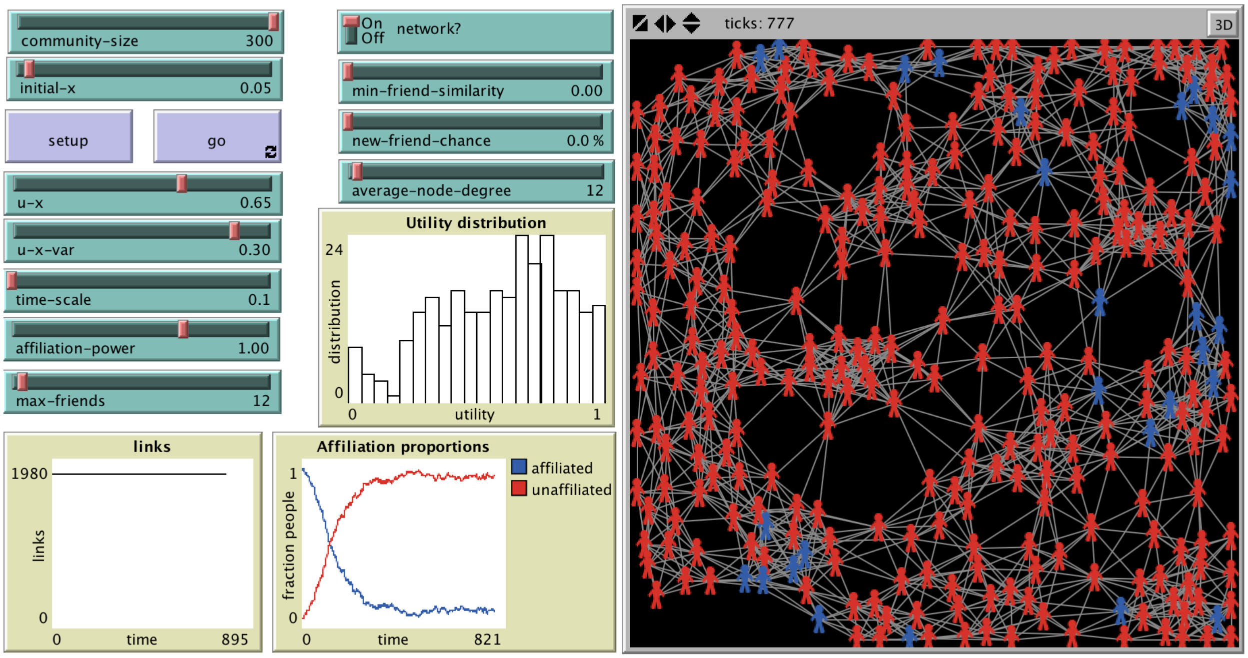

Figure 6: The continuous model results are not robust to large variation in affiliation utility. The population stabilizes with a small but significant proportion of the population still religiously affiliated.

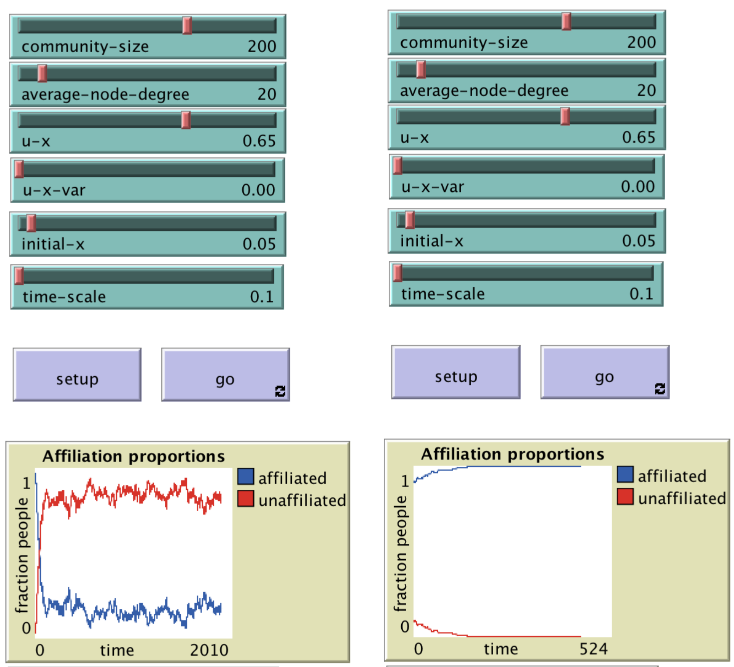

Figure 7: The continuous model results are not robust to changes in the exponent a of the power law switching probability. For $a < 1$ the population can coexist (left, $a = 0.6$). The early dynamics do not contradict the real-world data, so this is a plausible outcome. For $a > 1$ the affiliated can be the sole survivors (right, $a = 1.4$). The early dynamics are inconsistent with real-world data, so this case is not supported by the model. All simulations were performed with an all-to-all network and uniform utility.

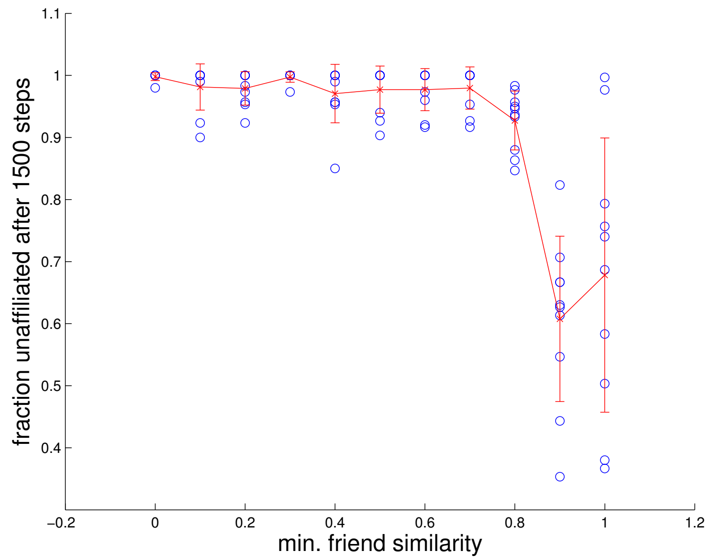

Figure 8: For a constant distribution of u_x centered about 0.65 with standard deviation 0.2, the fraction of unaffiliated individuals at equilibrium generally decreases as the minimum friend similarity increases. These simulations were run with no chance to make new friends, so high minimum friend similarity caused many agents to break away from the network entirely. Blue dots indicate individual simulations, red x's indicate the average of the 10 simulations, and the red error bars indicate $\pm$ one standard deviation.

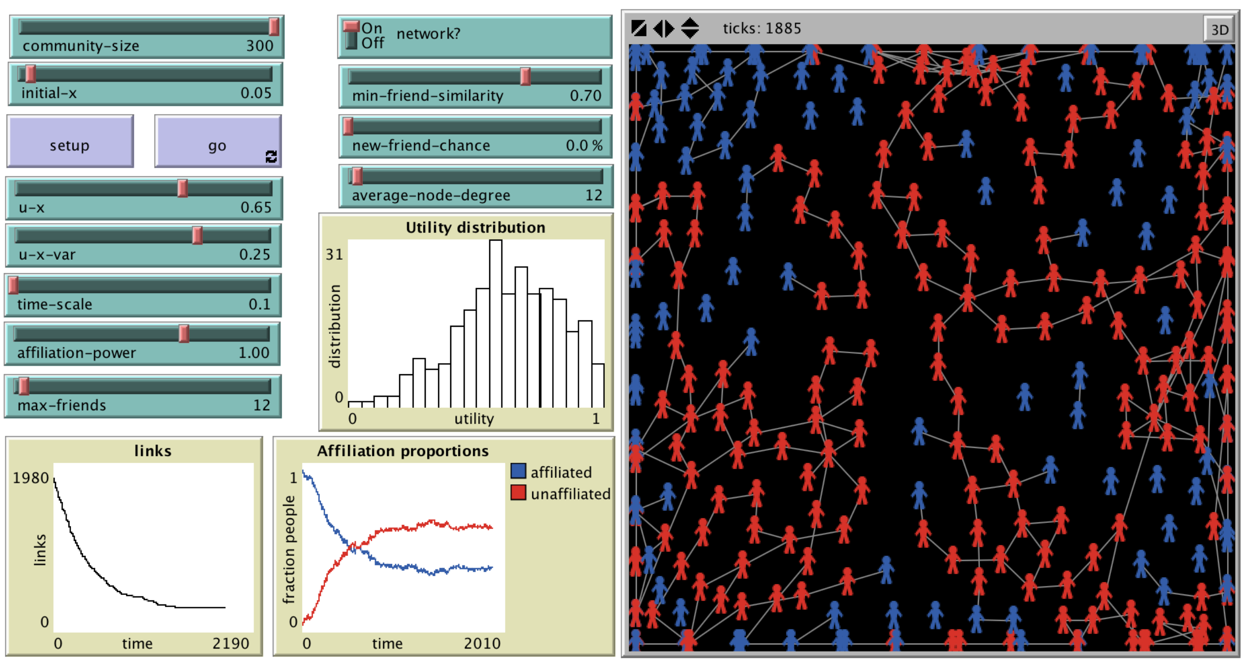

Figure 9: When the agents want to be friends with at least 70% similarly affiliated agents but can't make new friends, the two groups can coexist. At equilibrium, affiliated agents often have either formed their own separate clusters or disconnected from the network entirely.

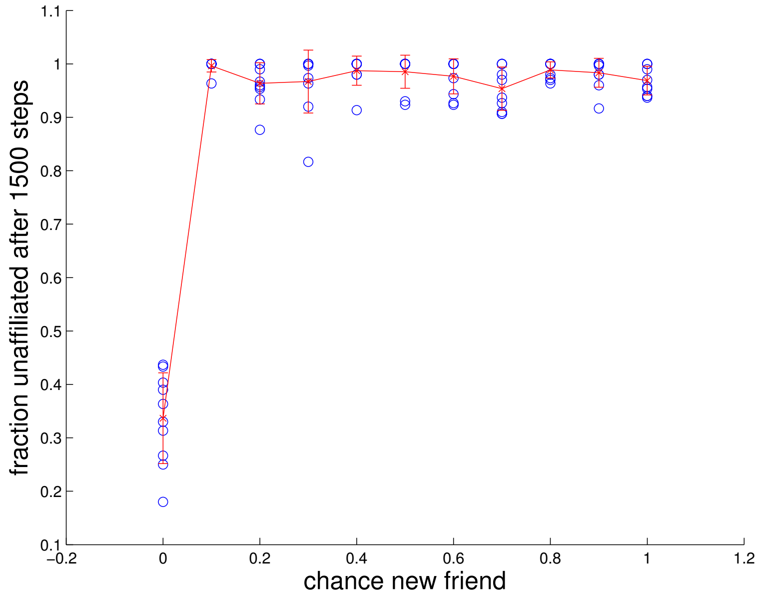

Figure 10: For a constant distribution of u_x centered about 0.65 with standard deviation 0.2, the fraction of unaffiliated individuals at equilibrium is relatively constant for NEW-FRIEND-CHANCE exceeding 0. Blue dots indicate individual simulations, red x's indicate the average of the 10 simulations, and the red error bars indicate $\pm$ one standard deviation.

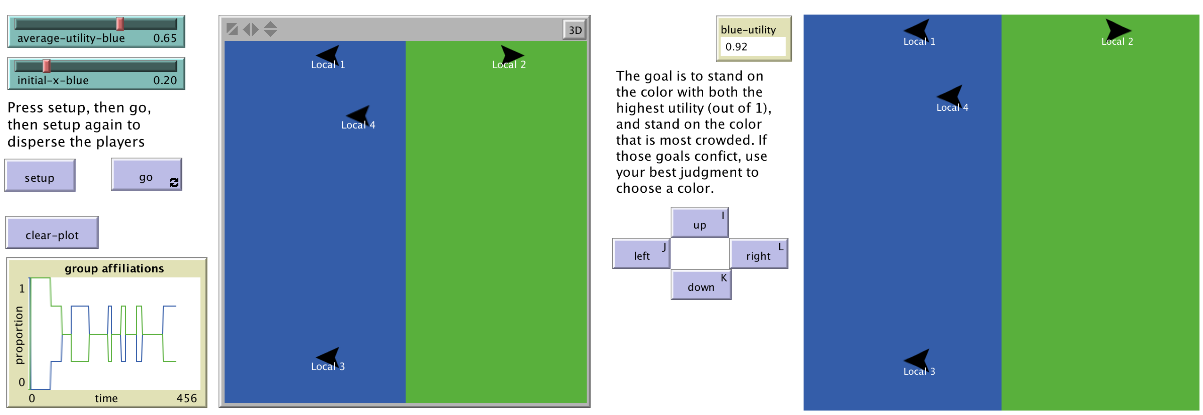

Figure 11: Generic social group competition model using HubNet where users choose a group to join based on the current membership and utility of the group. The left panel is the observer view, and the right panel is the player view.

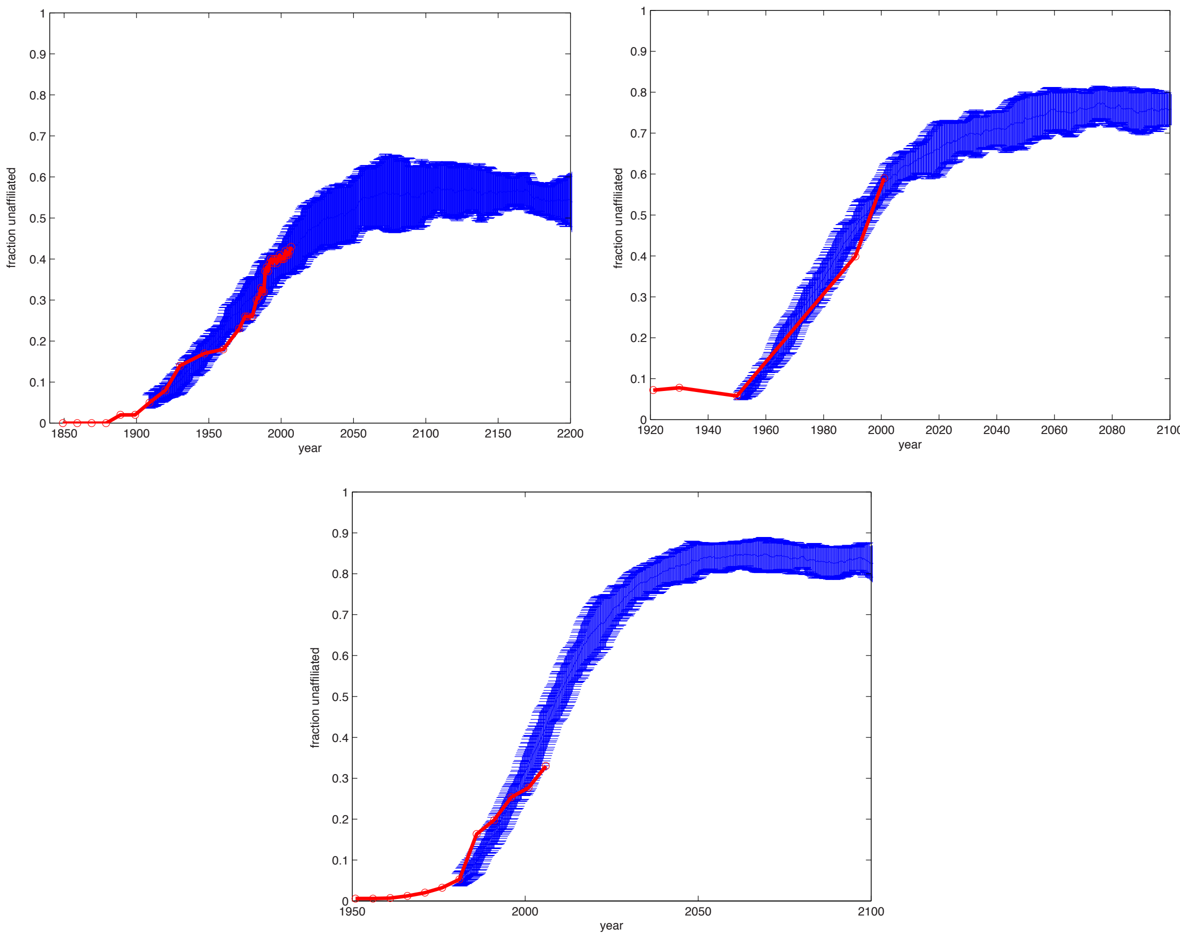

Figure 12: Model predictions for three illustrative datasets: Netherlands (top left), Czech Republic (top right), and New Zealand (bottom). I vary only the average u_x when fitting the data to the model and hold all other parameters constant across all datasets. For these simulations, I took community size 300, utility standard deviation 0.3, new friend chance 0.5, minimum friend similarity 0.7, and affiliation power 1. Note that all simulations begin with 0.05 initially unaffiliated agents due to small community size; if I begin the simulation with the real unaffiliated proportion, no agents would be initially unaffiliated. Red lines are real census data, and blue error bars indicate $\pm$ one standard deviation from the mean of 10 simulations.